# GCSE Maths – Geometry and Measures

## Area and Perimeter of 2D Shapes

Notes

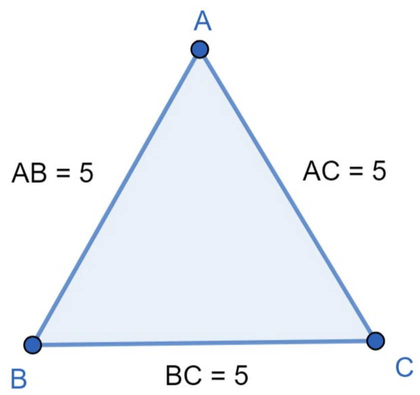
WORKSHEET

# Perimeter of 2D Shapes

A 2D shape is a shape that is **flat**, such as a triangle, square or rectangle. These shapes have only **two dimensions**: length and width.

The **perimeter** of a 2D shape is the length, or distance, **around the outside** of the shape. To calculate the perimeter, we need to know the length of each side. We then add together the length of each side.

For example, consider the following triangle:

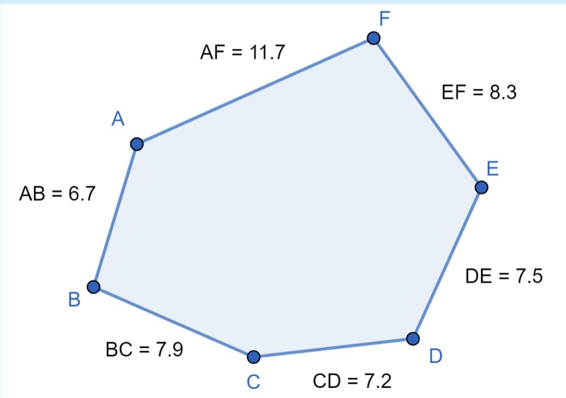



The length of each side of the triangle is 5 cm. To find the perimeter, we must **add together** all the side lengths:

$$Triangle\ Perimeter = 5 + 5 + 5 = 15 \text{ cm}$$

The perimeter of this triangle is 15 cm.

**Example:** Calculate the perimeter of this hexagon. The lengths are in centimetres (not to scale).



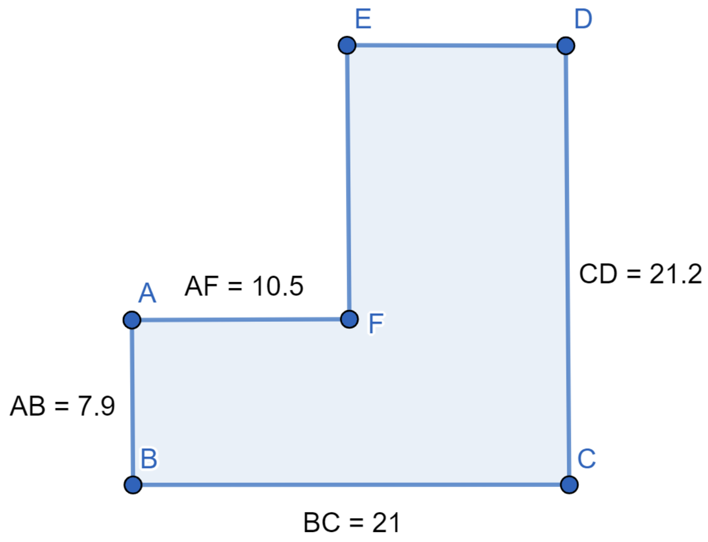

*Find the perimeter by adding together the length of each side:*

$$Perimeter = 6.7 + 7.9 + 7.2 + 7.5 + 8.3 + 11.7 = \mathbf{49.3\ cm}$$

## Composite Shapes

Composite shapes are shapes made from **two or more 2D shapes**. Often, we can use information we already know to calculate unknown side lengths.

For example, look at the following composite shape, where all lengths are given in centimetres:




We need to calculate the length of **EF** and **ED** and calculate the perimeter of the whole shape. To do this, we need to use the information we have already:

- We can see that the length of **BC** is equal to the length of **AF + ED**. As we know that BC is 21 cm, we can subtract find ED as follows:

$$\boldsymbol{ED} = \boldsymbol{BC} - \boldsymbol{AF} = 21 - 10.5 = 10.5 \text{ cm}$$

  So, **ED** is 10.5 cm.

- The length **CD** is equal to **AB + EF**. Subtracting AB from CD tells us the length of EF.

$$\boldsymbol{EF} = \boldsymbol{CD} - \boldsymbol{AB} = 21.2 - 7.9 = 13.3 \text{ cm}$$

- Now that we know the length of each side, we can add them all together to find the perimeter of the whole shape.

$$Perimeter = 7.9 + 21 + 21.2 + 10.5 + 13.3 + 10.5 = 84.4 \text{ cm}$$

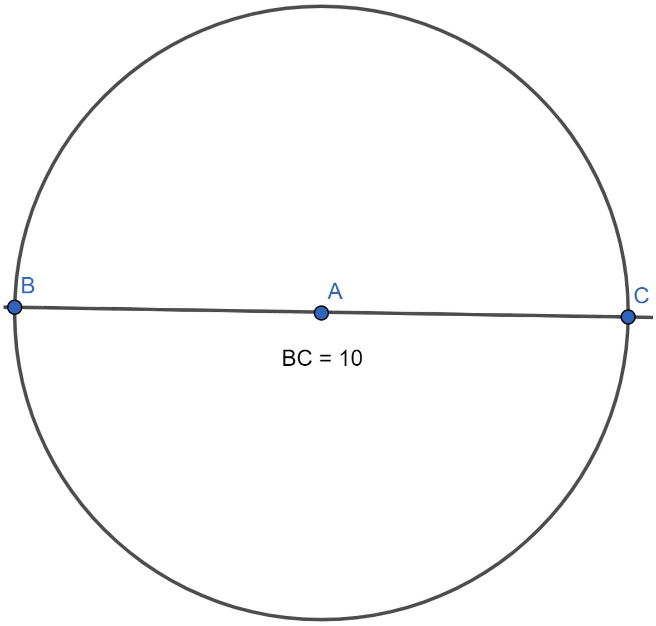
## Perimeters of Circles

When working with circles, the perimeter is called the **circumference**. The circumference can be found as followed:

$$Circumference = \pi \times Diameter$$

The diameter of a circle is the **distance across the circle**. The radius is the distance from the centre of the circle to the circumference. So, the diameter is two times the radius.

Consider the following circle:




Point **A** is the centre of the circle. The line **BC**, which passes through the midpoint of the circle, is 10 cm long. This is the **diameter** of the circle.

To calculate the circumference of this circle, we need to multiply the diameter by $\pi$.

$$Circumference = \pi \times Diameter = \pi \times 10 = 31.42 \text{ cm } (2\ d.p.)$$

**Example:** A circle has a radius of 9 cm. Calculate the circumference of this circle to 2 decimal places.

*Recall the formula for the circumference of a circle:*

$$Circumference = \pi \times Diameter$$

*To find the diameter, we need to know the distance across the circle. This is double the radius. Therefore,*

$$Diameter = 2 \times radius = 2 \times 9 \text{ cm} = 18 \text{ cm}$$

*Now calculate the circumference:*

$$Circumference = \pi \times Diameter = \pi \times 18 = 56.55 \text{ cm } (2\ d.p.)$$

# Area of 2D Shapes

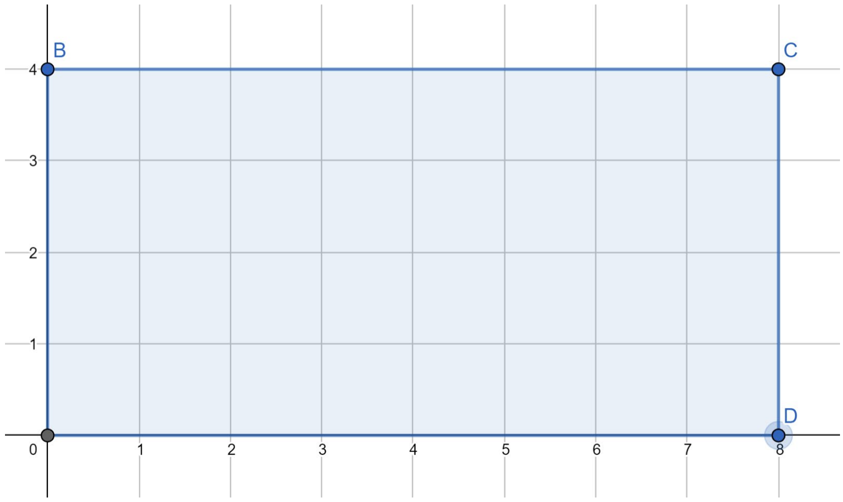
The area of a 2D shape is a measure of **how much space is contained within** the shape. The method for calculating the area of a 2D shapes depends on the type of shape.

## Areas of Rectangles

Calculating the area of a rectangle is relatively straightforward. Remember, a square is a type of rectangle, so the same formula holds for squares:

$$Area\ of\ Rectangle = Length \times Width$$

We can visualise this on graph paper.
For example, let's calculate the area of the following square:




The area is equal to the number of units squares within the shape. We could count the squares, or we could multiply the length of the shape by the width:

$$Area\ of\ Rectangle = Length \times Width = 8 \times 4 = 32\ \text{units}^2$$

If we were to count the squares within the rectangle, we would find there are 32 squares.

One thing we must remember when calculating area is to use the correct **units**. Since we have multiplied one dimension (length) by another (width), we write squared ($^2$) after the units, such as $\text{cm}^2$, $\text{mm}^2$, $\text{units}^2$.

**Example:** A square has sides of length 5 cm. A rectangle has a length of 4 cm and a width of 6 cm. Which shape has the larger area?

*Calculate the area of each shape:*

$$Area\ of\ Square = 5 \times 5 = 25\ \text{cm}^2$$
$$Area\ of\ Rectangle = 4 \times 6 = 24\ \text{cm}^2$$

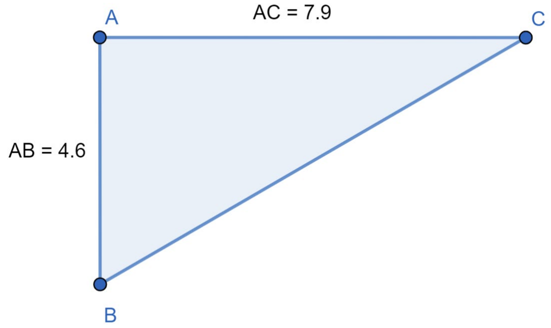
*The square has the larger area.*

## Area of Triangles

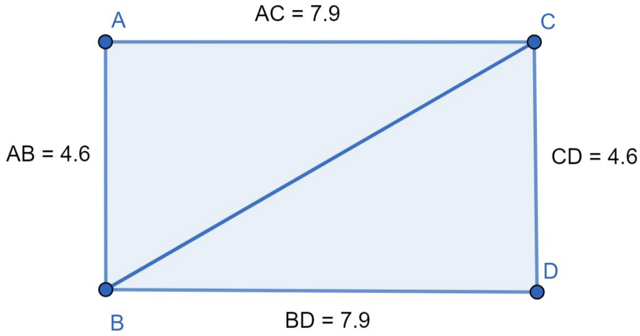
The formula for the area of a triangle is:

$$Area\ of\ Triangle = \frac{Base \times Height}{2}$$

To visualise this formula, look at the following triangle:




We can imagine this triangle as being **half of a rectangle**:




Two of these triangles together make a rectangle with a length of 4.6 and a width of 7.9. We already know how to calculate the area of a rectangle:

$$Area\ of\ Rectangle = Length\ \times Width = 4.6 \times 7.9 = 36.34\ \text{units}^2$$

Since it took **two triangles** to make this rectangle, the area of one triangle is equal to **half** of the area of the rectangle:

$$Area\ of\ Triangle = 36.34 \div 2 = 18.17\ \text{units}^2$$

Returning to the formula for the area of a triangle, $Triangle\ Area = \frac{Base \times Height}{2}$, we are essentially treating the triangle as one half of a rectangle.

**Example:** A triangle has a base of 7 cm and a height of 6 cm. Calculate the area of this triangle.

*Recall the formula for the area of a triangle:*

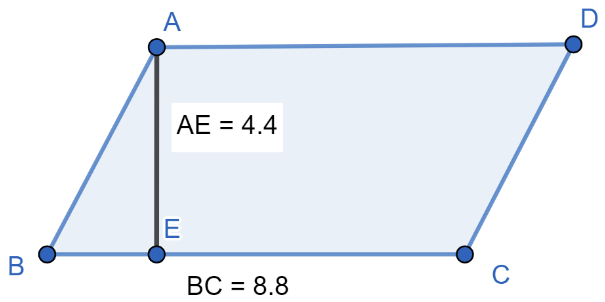
$$Triangle\ Area = \frac{Base \times Height}{2} = \frac{7 \times 6}{2} = 21\ \text{cm}^2$$

**Area of Parallelograms**

To calculate the area of a parallelogram, we use the following formula:

$$Area\ of\ Parallelogram = Base \times Height$$

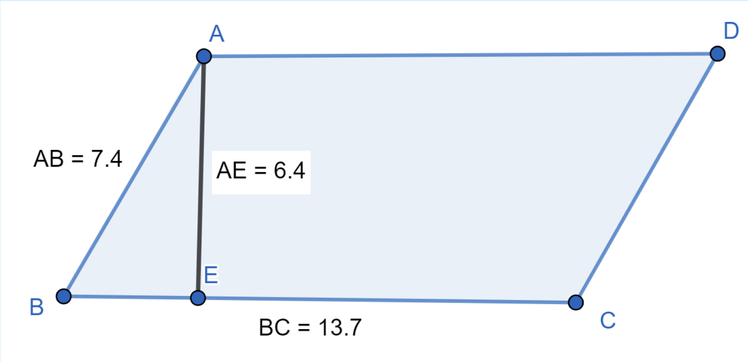
For this calculation, we must ensure we are using the **perpendicular height** of the parallelogram, not the length of the sloped side.




To work out the area of the parallelogram above, we compute:

$$Area\ of\ Parallelogram = Base \times Height = 8.8 \times 4.4 = 38.72\ \text{units}^2$$

**Example:** Calculate the area of the following parallelogram




*In the formula for parallelogram area, make sure to use the perpendicular height, not the length of the sloped side:*

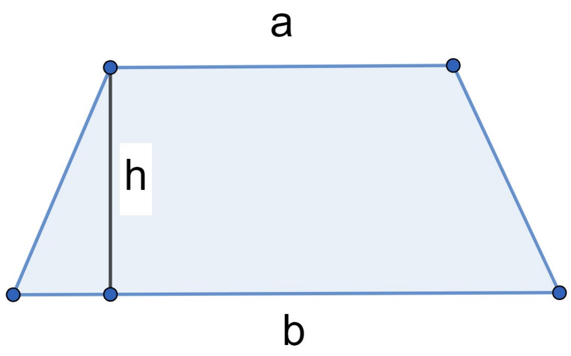
$$Area\ of\ Parallelogram = Base \times Height = 13.7 \times 6.4 = 87.68\ \text{units}^2$$

## Area of Trapezia

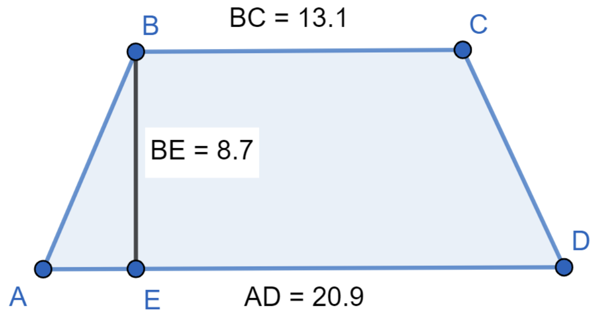
To calculate the area of a trapezium, use the following formula:

$$Area\ of\ Trapezia = \frac{1}{2}(a + b) \times h$$

where $a$ and $b$ are the lengths of the parallel sides and $h$ is the height:




For example, consider the following trapezium:




To calculate the area of the shape above, we can substitute the known values into the formula above:

$$Area\ of\ Trapezia = \frac{1}{2}(a + b) \times h = \frac{1}{2}(13.1 + 20.9) \times 8.7 = 147.9\ \text{units}^2$$

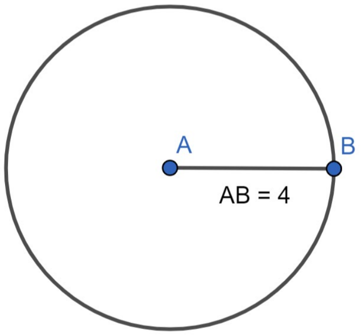
## Area of Circles

The area of a circle is calculated using the formula

$$Area\ of\ Circle = \pi r^2$$

where $r$ is the radius of the circle.

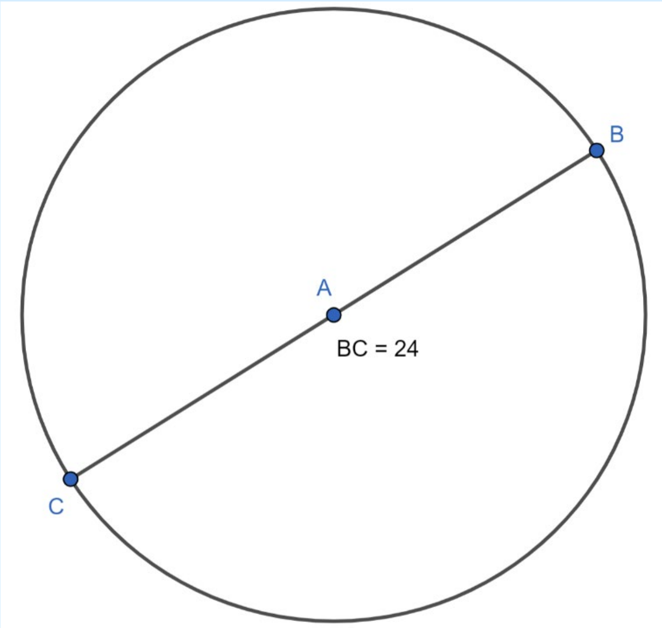
For example, consider the following circle:




The radius of this circle is 4 units long. Therefore, to calculate the area we calculate:

$$Area\ of\ Circle = \pi r^2 = \pi \times 4^2 = 50.27\ \text{units}^2\ (2\ d.p.)$$

**Example:** Calculate the area of the following circle




*We have been given the diameter of the circle, but to calculate the area, we need the radius.*
*To find the radius, divide the diameter by 2:*

$$Radius = Diameter \div 2 = 24 \div 2 = 12$$

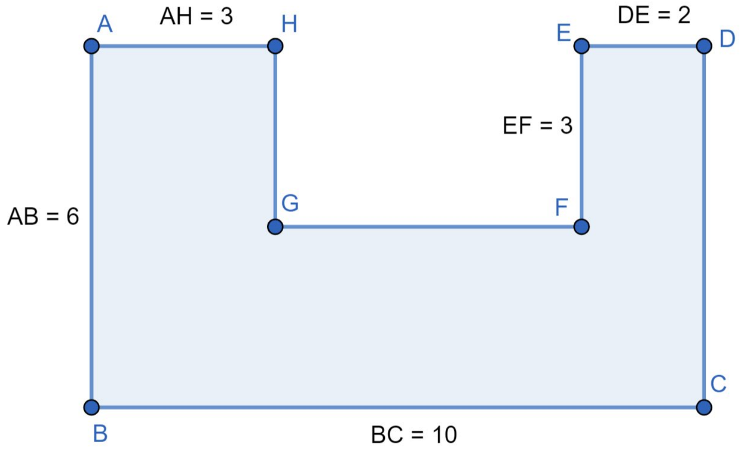
*Now we can substitute the radius into the formula for the area of a circle:*

$$Area\ of\ Circle = \pi r^2 = \pi \times 12^2 = 452.39\ \text{units}^2$$

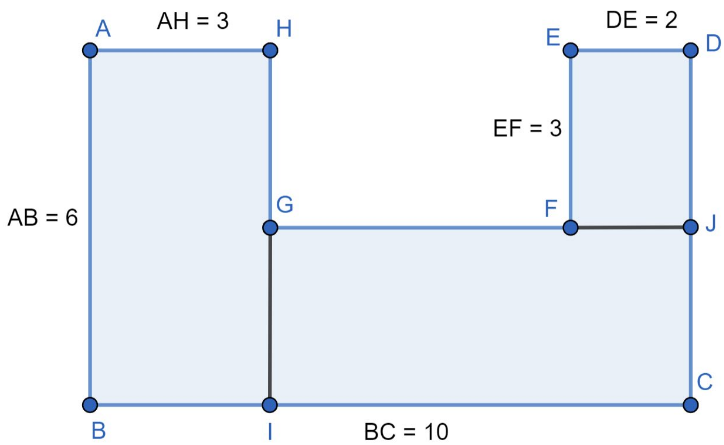
## Area of Composite Shapes

A composite shape is a shape made up of two or more 2D shapes. To calculate the area of a composite shape, we **add together** the areas of the shapes we know. In some cases, we can **section** the composite shape into **smaller shapes** of which we can work out the area.

For example, consider the following composite shape:




To work out the area of this shape, we can split it up into rectangles, like so:




- The rectangle **ABIH** has a length of 6 and a width of 3.

$$Area\ \boldsymbol{ABIH} = Length \times Width = 6 \times 3 = 18 \text{ units}^2$$

- The rectangle **CJGI** has a length of 3 – we know this because we can subtract EF from AB – and a width of 7 $(BC - AH)$.

$$Area\ \boldsymbol{CGJI} = Length \times Width = 7 \times 3 = 21 \text{ units}^2$$

- The rectangle **FJDE** has a length of 3 and a width of 2.

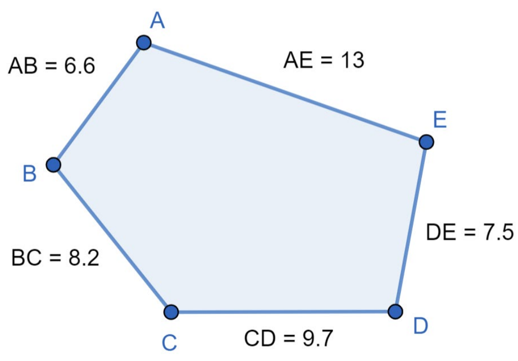
$$Area\ \boldsymbol{FJDE} = Length \times Width = 3 \times 2 = 6 \text{ units}^2$$

We then add together the area of each smaller shape to find the total area:

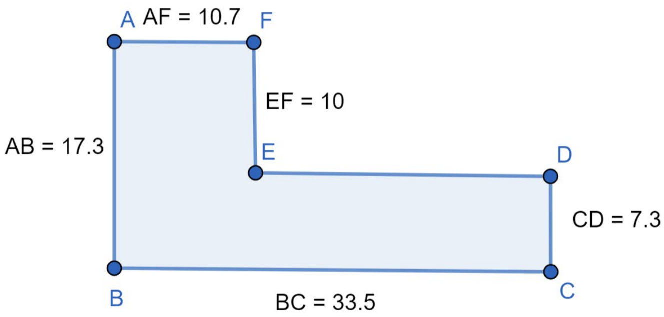
$$Total\ Area = 18 + 21 + 6 = \mathbf{45\ units^2}$$

## Area and Perimeter of 2D Shapes – Practice Questions

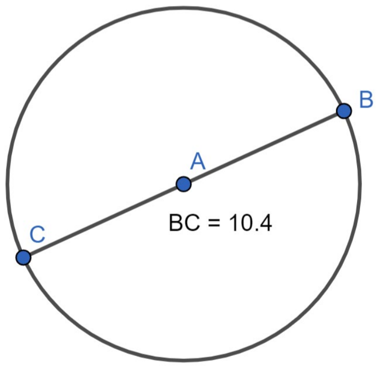
1. Calculate the perimeter of the following pentagon:




2. Calculate the perimeter of the following composite shape:




3. Calculate the circumference of the following circle:




*Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.*